Tribhuvan University
Institute of Science and Technology
Model Question

Bio Mathematics	Full Marks: 100
MAT 407	Pass Marks: 35
Level B.Sc. 4th Year	Time: 3 Hours.

Attempt all questions.

1. (a) The population of elephants in Chitwan National park is 200, find the population after 5 years, if the growth rate is 5% per year. [5]

   (b) In the model $\Delta P = 1.3P(1 - P/10)$, what values of P will cause ΔP to be positive? Negative? Why does this matter biologically? [5]

2. (a) What are the equilibrium points of $P_{t+1} = P_t + 0.7P_t(1 - P_t/10)$ and discuss their stability. [5]

   (b) Solve the logistic differential equation [5]

$$\frac{dN}{dt} = rN[1 - N/K], \; N(0) = N_0$$

3. In a study of insect population, the individuals progress from egg to larva over one step, and larva to adult over another. Finally, adults lay eggs and die in one more time step. Formulate the problem, for which 4% of the eggs survive to become larva , only 39% of the larva make to adulthood, and adults on average produce 73 eggs each. If E_t, L_t and A_t represent the number of eggs, number of larvae and the number of adults respectively, (a) prove that [4+ 4 + 2]

$$A_{t+1} = (0.39)(0.04)(73)A_t = 1.1388A_t$$

   (b) Express the model int he form $x_{t+1} = Px_t$

4. For the matrix

$$P = \begin{pmatrix} 0.9925 & 0.0125 \\ 0.0075 & 0.9875 \end{pmatrix}$$

   find eigenvalues and eigenvectors.

5. Derive the predator-prey model:

$$P_{t+1} = P_t + rP_t(1 - P_t/K) - sP_tQ_t,$$

$$Q_{t+1} = Q_t - uQ_t + vP_tQ_t$$

   where r, s, u, v, K are positive constants and $u < 1$, and all the symbols have their usual meanings. Find their equilibrium points. [6 + 4]

Or

For the predator - prey model

$$P_{t+1} = P_t(1 + 1.3(1 - P_t))0.5P_tQ_t,$$
$$Q_{t+1} = 0.3Q_t + 1.6P_tQ_t$$

(a) compute the Jacobian matrix (b) Evaluate the Jacobian matrix at the equilibrium points. [4+ 6]

6. Consider the 20-base sequence $AGGGATACATGACCCATACA$. [10]

   (a) Use the first five bases to estimate the four probabilities p_A, p_G, p_C, and p_T .

   (b) Repeat part (a) using the first 10 bases.

   (c) Repeat part (a) using all the bases.

7. For n terminal taxa, the number of unrooted bifurcating trees is

$$1 \cdot 3 \cdot 5 \cdots (2n - 5) = \frac{(2n - 5)!}{2^{n-3}(n - 3)!}$$

Make a table of values and graph this function for $n \leq 10$. [10]

8. Find the probability that the progeny of $DdWw \times ddWw$ is dwarf with round seeds. [10]

Or

Assuming births of each sex are equally likely, a two-child family may have 4 outcomes in the sexes of the children. (a) List the outcomes and give the probability of each. (b) What is the probability that at least one child is a female? (c) What is the probability that the youngest child is a female? (d) What is the conditional probability that the youngest child is a female, given that at least one child is a female? (e) What is the conditional probability that at least one child is a female, given that the youngest child is a female?

9. Derive the SIR model. Find the threshold value. [5+ 5]

Or

Derive SI and SIS models. Solve for all equilibria (S^*, I^*). [7+3]

10. Plot the three points $(1, 1), (0, 3)$, and $(1, 4)$. Then, find the least squares, best-fit line for them. Draw a graph of the line to your plot. [10]

Or

Drug levels in the bloodstream are typically observed to decay exponentially with time from the administration of a dose. A difference equation model that describes this (and gives further reason to try to fit the data of Table

t (day)	0	1	2	3	4
y (mg/l)	200	129	-	58	33

to an exponential curve) is $y_{t+1} = (1-r)y_t$, where r is the percentage of the drug that is absorbed by tissue or broken down by metabolization during one time step. (a) If the initial amount of the drug is y_0, explain why this model leads to $y_t = y_0(1 - r)^t$. (b) Letting $k = \ln(1 - r)$ and $a = y_0$, show this is equivalent to $y_t = ae^{kt}$. (c) Explain why $0 < r < 1$ for this model, and then why $k < 0$.

MODEL QUESTION
Tribhuvan University

Four year Bachelor Level/ Science & Tech./ Year IV — Full Marks: 100
Mathematical Economics (MAT. 408) — Pass Marks: 35
NEW COURSE — Time: 3 Hrs

Attempt ALL the questions.

1. (a) Define market equilibrium. Write two-commodity market model and extract the equilibrium condition of the model. [1+5]

   (b) Extract the equilibrium solution of the following model. [4]

$$\begin{aligned} Q_{d1} &= 10 - 2P_1 - 2P_1 + P_2 \\ Q_{s1} &= -2 + 3P_1 \\ Q_{d2} &= 15 + P_1 - 2P_1 - P_2 \\ Q_{s2} &= -1 \end{aligned}$$

2. Consider the situation of a mass layoff where 1200 people become unemployment and now begin a job search. In this case there are two states: employed (E) and unemployed (U) with an initial vector $x_0' = [E\ U] = [0\ \ 1200]$. Suppose that in any given period an unemployed person will find a job with probability 0.7. Additionally, people who find themselves employed in any given period may lose their job with a probability of 0.1.

   (a) Set up the Markov transition matrix for this problem.

   (b) What will be the number of unemployment people after (i) 2 periods; (ii) 10 periods?

   (c) What is the steady-state level of unemployment? [5+4+1]

3. (a) Define marginal-cost and average-cost. Given the total-cost function $C = Q^3 - 5Q^2 + 12Q + 75$, write out a variable-cost(VC) function. Find the derivative of the VC function and interpret the economic meaning of derivative.

   (b) Write the economical interpretation of partial derivatives. Use Jacobian determinants to test the existence of functional dependence between the following paired functions.
   $y_1 = 3x_1^2 + x_2; y_2 = 9x_1^4 + 6x_1^2(x_2 + 4) + x_2(x_2 + 8) + 12$ [5+5]

4. What are the main assumptions of the IS-LM national-income model? Derive the slope of IS and LM curves, write the equilibrium identities of the curves. Hence, find the comparative-static derivatives of the model. [1+3+2+4]

OR

Define differentials and point elasticity. Let the equilibrium condition for national income be $S(Y) + T(Y) = I(Y) + G_0 \quad (S', T', I' > 0 : S' + T' > I')$ where $S, Y, T, I,$ and G stand for saving, national income, taxes, investment, and government expenditure, respectively. All derivatives are continuous.

   (a) Interpret the economic meaning of the derivatives S', T', and I'.

   (b) Check whether the conditions of the implicit-function theorem satisfied. If so, write the equilibrium identity.

   (c) Find $\frac{dY^*}{dG_0}$ and discuss its economic implications. [2+5+3]

5. Formulate the wine storage problem and extract the maximization conditions for the problem. [2+8]

OR

Write the first-order and second-order conditions for extremum of more than two variables. Find the extreme values of $Z = -x_1^3 + 3x_1x_3 + 2x_2 - x_2^2 - 3x_3^2$. Check whether they are maxima or minima by the determinantal test. [4+6]

6. A two-product firm faces the following demand and cost functions: $Q_1 = 40 - 2P_1 - P_2$; $Q_2 = 35 - P_1 - P_2$; $C = Q_1^2 + 2Q_2^2 + 10$

   (a) Find the output levels that satisfy the first-order condition for maximum profit.

   (b) Check the second-order sufficient condition. Can you conclude that this problem possesses a unique absolute maximum?

   (c) Find the maximum profit? [4+4+2]

7. (a) Find the extremum of the optimization problem $z = xy$, subject to $x + y = 6$ using the Lagrange-multiplier method.

   (b) Check the optimality of the programming $z = xy$, subject to $x + 2y = 2$ using the bordered Hessian. [5+5]

8. State the Kuhn-Tucker sufficiency conditions for concave programming. Solve the following problem applying the Kuhn-Tucker conditions. [2+8]

$$\begin{aligned} \text{Minimize } C &= (x_1 - 4)^2 + (x_2 - 4)^2 \\ \text{subject to } & 2x_1 + 3x_2 \geq 6 \\ & -3x_1 - 2x_2 \geq -12 \\ & x_1, x_2 \geq 0 \end{aligned}$$

OR

State and establish the Roy's identity. Consider a consumer with the utility function $U = xy$, who faces a budget constraint of B and is given price P_x and P_y. Does the Roy's identity hold for the following choice problem? [5+5]

$$\begin{aligned} \text{Maximize } U &= xy \\ \text{subject to } & P_x x + p_y y = B \end{aligned}$$

9. (a) Find the present value of continuous revenue flow lasting for y years at the constant rate of D dollars per year and discounted at the rate of r per year.

   (b) Find the present value of a perpetual cash flow of:

      i. \$1450 per year, discounted at $r = 5\%$.

      ii. \$2460 per year, discounted at $r = 8\%$. [5+5]

10. Write the basic premises of Domar's growth model and extract the solution to the model. [4+6]

OR

Let the demand and supply functions

$$\begin{aligned} Q_d &= 40 - 2P - 2P' - P'' \\ Q_s &= -5 + 3P \end{aligned}$$

with $P(0) = 12$ and $P'(0) = 1$

   (a) Find the price path, assuming market clearance at every point of time.

   (b) Is the time path convergent? Mention its fluctuation. [7+3]